

Chapter 30

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance $r = 6.1$ m from the wire, is given by

$$B = \frac{\mu_0 |i|}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100 \text{ A})}{2\pi(6.1 \text{ m})} = 3.3 \times 10^{-6} \text{ T} = 3.3 \mu\text{T}.$$

(b) This is approximately one-sixth the horizontal component of the Earth's field. It will affect the compass reading.

11. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 |i| \sin \phi}{4\pi r^2} dx$$

where ϕ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x . Replacing r with $\sqrt{x^2 + R^2}$ and $\sin \phi$ with $R/r = R/\sqrt{x^2 + R^2}$, we integrate from $x = -L/2$ to $x = L/2$. The total field has a magnitude of

$$B = \frac{\mu_0 |i| R}{4\pi} \int_{-L/2}^{L/2} \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 |i| R}{4\pi} \frac{1}{R^2} \frac{x}{(x^2 + R^2)^{1/2}} \Big|_{-L/2}^{L/2} = \frac{\mu_0 |i|}{2\pi R} \frac{L}{(L^2 + 4R^2)^{1/2}}.$$

If $L \gg R$, then R^2 in the denominator can be ignored and

$$B = \frac{\mu_0 |i|}{2\pi R}$$

is obtained. This is the field of a long straight wire. For points very close to a finite wire, the field is quite similar to that of an infinitely long wire.

21. Let the x axis point to the right and the y axis point up on the page. The wire on the top will produce a magnetic field with a positive y -component and a positive x -component. The wire on the bottom will produce a magnetic field with a positive y -component and a negative x -component. Since the wires carry the same current and are equidistant from the point P , the x -components of the fields add to zero. With θ as the angle between x axis and the line between P and one of the wires, the y -components add to give the magnitude of the field:

$$\begin{aligned} B &= 2 \frac{\mu_0 |i|}{2\pi \left(\left(\frac{d_1}{2} \right)^2 + d_2^2 \right)^{1/2}} \cos \theta = \frac{\mu_0 |i|}{\pi \left(\left(\frac{d_1}{2} \right)^2 + d_2^2 \right)^{1/2}} \frac{d_2}{\left(\left(\frac{d_1}{2} \right)^2 + d_2^2 \right)^{1/2}} \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.00 \text{ A})(4.00 \text{ m})}{\pi \left((3.00 \text{ m})^2 + (4.00 \text{ m})^2 \right)} = 2.56 \times 10^{-7} \text{ T}. \end{aligned}$$

27. Each wire produces a field with magnitude given by $B = \mu_0 |i|/2\pi r$, where $r = a/\sqrt{2}$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

$$B = 4 \frac{\mu_0 |i|}{\sqrt{2}\pi a} \cos 45^\circ = \frac{2\mu_0 |i|}{\pi a} = \frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(20 \text{ A})}{\pi(0.20 \text{ m})} = 8.0 \times 10^{-5} \text{ T}.$$

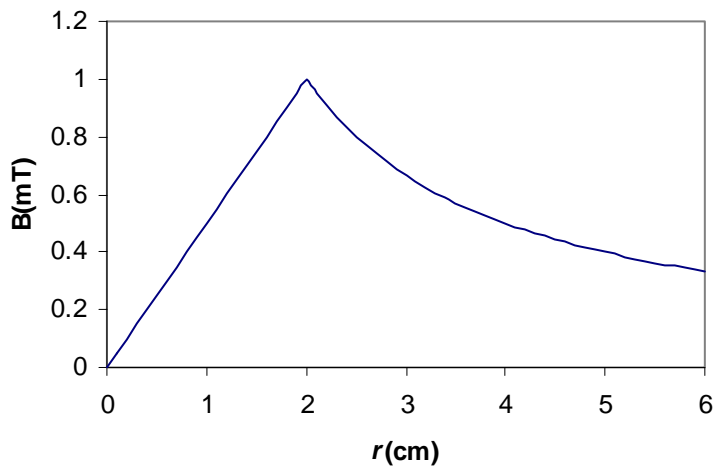
The field points upward on the page.

33. (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i^{\text{enc}} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(-2.0 \text{ A}) = -2.5 \times 10^{-6} \text{ T} \cdot \text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i^{\text{enc}} = 0$.

34. We use Eq. 30-22 for the B -field inside the wire and Eq. 30-19 for that outside the wire.



38. (a) We use Eq. 30-22, with the substitution $J = |i|/\pi R^2$. The magnetic field evaluated at $r = 2.0 \text{ mm}$ has a magnitude of

$$B = \frac{1}{2} \mu_0 J r = \frac{1}{2} (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (100 \text{ A/m}^2) (0.0020 \text{ m}) = 1.3 \times 10^{-7} \text{ T}.$$

(b) Here we use Eq. 30-19, with the substitution $J = |i|/\pi R^2$. The magnetic field evaluated at $r = 4.0 \text{ mm}$ has a magnitude of

$$B = \frac{\mu_0 J R^2}{2r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (100 \text{ A/m}^2) (0.0030 \text{ m})^2}{2(0.0040 \text{ m})} = 1.4 \times 10^{-7} \text{ T}$$