Chapter 27

4. The current in the circuit is $i = (150\ \text{V} - 50\ \text{V})/(3.0\ \Omega + 2.0\ \Omega) = 20\ \text{A}$. Knowing the current allows us to calculate the voltage difference between points *Q* and *P*.

$$\Delta V_{P \to Q} = -50\ \text{V} - (20\ \text{A})(3.0\ \Omega) = -110\ \text{V}.$$

Since the potential at point *P* is 100 V, the potential at point *Q* is -10 V.

6. (a) For each wire,

$$R_{\text{wire}} = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(0.200\ \text{m})}{\pi (1.00 \times 10^{-3}\,\text{m})^2} = 1.08 \times 10^{-3}\,\Omega.$$

The wires are in series with the 6.00 Ω resistor, so the equivalent resistance is $6.00\ \Omega + 2(1.08 \times 10^{-3}\,\Omega) = 6.0022\ \Omega$, where we are temporarily maintaining a couple too many digits in our answer. Ohm's law gives us the current through the circuit as $i = 12.0\ \text{V}/6.0022\ \Omega = 1.9993\ \text{A}$. The potential difference across the 6 Ω resistor is therefore $\Delta V = (1.9993\ \text{A})(6.00\ \Omega) = 11.996\ \text{V} \approx 12.0\ \text{V}$.

(b) The potential difference across each wire is

$$\Delta V_{\text{wire}} = (1.9993\ \text{A})(1.08 \times 10^{-3}\ \Omega) = 2.16\ \text{mV}.$$

(c) The resistor uses energy at a rate of

$$P = i^2 R = (1.9993\ \text{A})^2 (6.00\ \Omega) = 23.98\ \text{W} \approx 24.0\ \text{W}.$$

(d) Each wire uses energy at a rate of

$$P_{\text{wire}} = i^2 R_{\text{wire}} = (1.9993\ \text{A})^2 (1.08 \times 10^{-3}\ \Omega) = 4.32\ \text{mW}.$$

9. (a) and (b) Assume i_1 travels to the right, i_2 up and i_3 down. (These assumptions are confirmed if the answers come back positive. If one comes back negative, we guessed wrong.) We use the loop rule twice, clockwise around the outer loop and clockwise around the right loop:

$$(10\ \text{V}) - i_1 R_1 - i_3 R_3 = 0\ \text{V};$$

$$(5.0\ \text{V}) - i_2 R_2 - i_3 R_3 = 0\ \text{V}.$$

We use the junction rule $(i_3 = i_1 + i_2)$ to eliminate i_3 in the loop rule equations. Substituting the given data into the loop rule equations yields

$$10\ \text{V} = i_1 (8.0\ \Omega) + i_2 (4.0\ \Omega);$$

$$5.0\ \text{V} = i_2 (8.0\ \Omega) + i_1 (4.0\ \Omega).$$

Solving for i_1 and i_2, we find $i_1 = 1.25\ \text{A}.$ and $i_2 = 0\ \text{A}$. Therefore $i_3 = i_1 + i_2 = 1.25\ \text{A}.$ As assumed above, i_3 is traveling down in the picture.

11. (a) The resistors R_2, R_3 and R_4 are in parallel with each other, and that combination is in series with R_1. The equivalent resistance for this network is

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 100\ \Omega + \left(\frac{1}{50\ \Omega} + \frac{1}{50\ \Omega} + \frac{1}{75\ \Omega}\right)^{-1} = 118.75\ \Omega \approx 120\ \Omega.$$

(b) Based on our result from part (a), the current through the battery must be $i_B = \Delta V_B / R_{eq} = 6.0\ \text{V} / 118.75\ \Omega = 0.0505\ \text{A} \approx 5.1\ \text{A}.$ Therefore the current through R_1 is $i_1 = 0.051\ \text{A},$ while the potential difference across R_1 is $\Delta V_1 = i_1 R_1 = (0.0505\ \text{A})(100\ \Omega) = 5.05\ \text{V}.$ Each of the three resistors in parallel must have a potential difference of 0.95 V across it. Ohm's law then gives $i_2 = i_3 = 0.019\ \text{A}$ and $i_4 = 0.013\ \text{A}.$

19. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward. When the loop rule is applied to the lower loop (counterclockwise), the result is

$$\Delta V_{B2} - i_1 R_1 = 0\ \text{V}.$$

and when it is applied to the upper loop (counterclockwise), the result is

$$\Delta V_{B1} - \Delta V_{B2} - \Delta V_{B3} - i_2 R_2 = 0\,\text{V}.$$

The first equation yields

$$i_1 = \frac{\Delta V_{B2}}{R_1} = \frac{5.0\ \text{V}}{100\ \Omega} = 0.050\ \text{A}.$$

The second yields

$$i_2 = \frac{\Delta V_{B1} - \Delta V_{B2} - \Delta V_{B3}}{R_2} = \frac{6.0\ \text{V} - 5.0\ \text{V} - 4.0\ \text{V}}{50\ \Omega} = -0.060\ \text{A}.$$

The negative sign indicates that the current in R_2 is actually downward.

The potential difference between points *a* and *b* is $V_{a\to b} = -\Delta V_{B2} - \Delta V_{B3} = -9.0\ \text{V}.$

59. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let *i* be the current in either battery and take it to be positive to the left. According to the junction rule the current in *R* is 2*i* and it is positive to the right. The loop rule applied to either loop containing a battery and *R* yields

$$\mathscr{E} - ir - 2iR = 0\ \text{V} \quad \Rightarrow \quad i = \frac{\mathscr{E}}{r + 2R}.$$

The power dissipated in *R* is

$$P = (2i)^2 R = \frac{4\mathscr{E}^2 R}{(r+2R)^2}$$

We find the maximum by setting the derivative with respect to *R* equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\mathscr{E}^2}{(r+2R)^2} - \frac{16\mathscr{E}^2 R}{(r+2R)^3} = \frac{4\mathscr{E}^2 (r-2R)}{(r+2R)^3}.$$

The derivative vanishes (and *P* is a maximum) if $R = r/2$.

(b) We substitute $R = r/2$ into $P = 4\mathscr{E}^2 R/(r+2R)^2$ to obtain

$$P^{\max} = \frac{4\mathscr{E}^2 (r/2)}{\left[r + 2(r/2)\right]^2} = \frac{\mathscr{E}^2}{2r}.$$