Chapter 24

1. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\text{j}} = (1.40\ \text{m})^2\ \hat{\text{j}}$.

(a) $\Phi = \left(6.00\ \text{N/C}\right)\hat{\text{i}} \cdot (1.40\ \text{m})^2\ \hat{\text{j}} = 0\ \text{N} \cdot \text{m}^2/\text{C}.$

(b) $\Phi = \left(-2.00\ \text{N/C}\right)\hat{\text{j}} \cdot (1.40\ \text{m})^2\ \hat{\text{j}} = -3.92\ \text{N} \cdot \text{m}^2/\text{C}.$

(c) $\Phi = \left[\left(-3.00\ \text{N/C}\right)\hat{\text{i}} + \left(400\,\text{N/C}\right)\hat{\text{k}}\right] \cdot (1.40\ \text{m})^2\ \hat{\text{j}} = 0\ \text{N} \cdot \text{m}^2/\text{C}.$

(d) Since the electric field is uniform in this region, there can be no source charges inside the cube. Therefore the total flux through the surface of the cube is $\Phi^{\text{net}} = 0\ \text{N} \cdot \text{m}^2/\text{C}$.

3. We use Gauss' law to relate the net flux through the surface to the charge enclosed by the surface:

$$\Phi^{\text{net}} = \frac{q^{\text{enc}}}{\varepsilon_0} = \frac{1.8 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 2.0 \times 10^5\ \ \text{N} \cdot \text{m}^2/\text{C}.$$

6. Since the netting and the opening of the net form a closed surface, the net flux through the netting and opening combined must be zero. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus, the flux through the netting is $\Phi' = -\Phi = -\pi a^2 E$.

17. (a) The total charge is obtained by integrating the volume charge density over the volume:

$$Q = \int_V \rho(r)\,dV = \int_0^R \rho(r) \cdot 4\pi r^2 dr = \frac{4\pi\rho_s}{R}\int_0^R r^3 dr = \left(\frac{4\pi\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = \pi\rho_s R^3.$$

(b) For spherically symmetric charge distributions, Eq. 24-11 is

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q'(r)\right|}{r^2},$$

where $\left|q'(r)\right|$ is the function describing how the enclosed charge varies with distance from the center of the charge distribution. To determine how much charge is enclosed by a Gaussian sphere of radius *r*, integrate the volume charge density for the volume of the Gaussian sphere (we integrate over the dummy variable *r′* as opposed to *r* since *r* is used as an integration limit):

$$\left|q'(r)\right| = \int_V \left|\rho\left(r'\right)\right| dV' = \int_0^r \left|\rho\left(r'\right)\right| \cdot 4\pi r'^2 dr' = \frac{4\pi\left|\rho_s\right|}{R}\int_0^r r'^3 dr = \left(\frac{4\pi\left|\rho_s\right|}{R}\right)\left(\frac{r^4}{4}\right) = \frac{\pi\left|\rho_s\right| r^4}{R}.$$

Therefore, using $Q = \pi\rho_s R^3$,

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0 r^2}\frac{\pi\left|\rho_s\right| r^4}{R} = k\frac{\pi\left|\rho_s\right|}{R}r^2 = k\frac{|Q|}{R^4}r^2.$$

19. For spherically symmetric charge distributions, Eq. 24-11 is

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q'(r)\right|}{r^2},$$

where $\left|q'(r)\right|$ is the function describing how the enclosed charge varies with distance from the center of the charge distribution.

(a) For $r < a,\ \left|q'(r)\right| = q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = q\frac{r^3}{a^3},$ so $\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{qr}{a^3}.$

The direction is radially outward.

(b) For $a < r < b,\ \left|q'(r)\right| = q,$ so $\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$

The direction is radially outward.

(c) For $b < r < c,$ since the shell is conducting, the electric field inside it is zero.

(d) For *r* > *c*, $\left|q'(r)\right| = 0$ (charge *q* is inside the shell cavity and charge –*q* is on the shell). Therefore, $E = 0.$

(e) The inner surface of the shell must carry charge $-q$ in order that zero charge be enclosed by a Gaussian surface with $b < r < c.$ Since the net charge of the entire shell is $-q$, and that is the same charge carried on the inner surface, the outer shell must carry zero charge.

20. For spherically symmetric charge distributions, Eq. 24-11 is

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q'(r)\right|}{r^2},$$

where $\left|q'(r)\right|$ is the function describing how the enclosed charge varies with distance from the center of the charge distribution.

For $r < a,\ \left|q'(r)\right| = 0,$ so $E(r < a) = 0.$

For $a < r < b, \left|q'(r)\right| = \rho V = \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right),$ so

$$E(a < r < b) = \frac{1}{4\pi\varepsilon_0}\frac{\rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)}{r^2} = \frac{\rho}{3\varepsilon_0}\left(r - \frac{a^3}{r^2}\right).$$

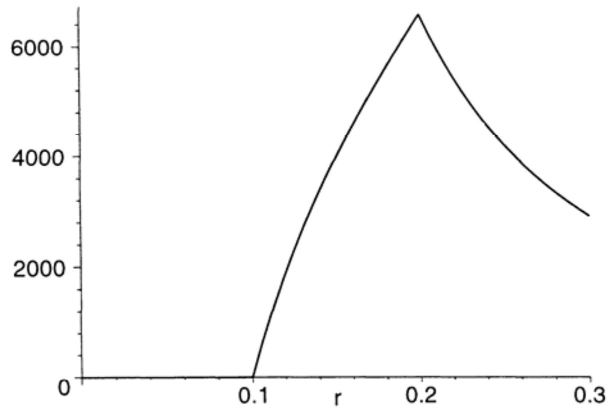
The direction is radially outward (since $\rho > 0.$)

For $r > b$, $\left|q'(r)\right| = \rho V = \rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)$, so

$$E(r > b) = \frac{1}{4\pi\varepsilon_0}\frac{\rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)}{r^2} = \frac{\rho}{3\varepsilon_0}\left(\frac{b^3 - a^3}{r^2}\right).$$

The direction is radially outward (since $\rho > 0$.)

These results are plotted next for *r* in meters from 0 to 0.30 m, using the values given in the text. The peak value of the electric field, reached at *r* = *b* = 0.20 m, is 6.6×10^3 N/C.



24. We imagine a cylindrical Gaussian surface *A* of radius *r* and length *h* concentric with the metal tube. Then by symmetry

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi r h E = \frac{q^{\text{enc}}}{\varepsilon_0}$$

(a) For *r* > *R*, $q^{\text{enc}} = \lambda h$, so $E(r) = \lambda/2\pi r\varepsilon_0$.

(b) For *r* < *R*, $q^{\text{enc}} = 0\ \text{C}$, so *E* = 0 N/C. The plot of *E* vs *r* is shown below. Here, the maximum value is

$$E^{\max} = \frac{|\lambda|}{2\pi R\varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^4\ \text{N/C}.$$