

Chapter 32

6. (a) The induced emf will be oriented in a direction such that it opposes any change in current. Since the current is to the right, and the emf is to the right, the current must be decreasing.

(b) From Eq. 32-3 we get

$$L = \left| \frac{\mathcal{E}_L}{di/dt} \right| = \frac{17 \text{ V}}{25 \times 10^3 \text{ A/s}} = 6.8 \times 10^{-4} \text{ H}.$$

19. (a) The flux over the loop cross section due to the current i in the wire is given by

$$\Phi^{\text{mag}} = \int_a^{a+b} B_{\text{wire}} l dr = \frac{\mu_0 i l}{2\pi} \int_a^{a+b} \frac{1}{r} dr = \frac{\mu_0 i l}{2\pi} \ln \left(1 + \frac{b}{a} \right).$$

Thus,

$$M = \frac{N \Phi^{\text{mag}}}{i} = \frac{N \mu_0 l}{2\pi} \ln \left(1 + \frac{b}{a} \right).$$

(b) From the formula for M obtained above,

$$M = \frac{(100)(4\pi \times 10^{-7} \text{ H/m})(0.30 \text{ m})}{2\pi} \ln \left(1 + \frac{8.0 \text{ cm}}{1.0 \text{ cm}} \right) = 1.3 \times 10^{-5} \text{ H}.$$

20. We use Eq. 32-22 to write the current as $i(t) = i^{\text{max}} (1 - e^{-t/\tau_L})$. Since $i(5.00 \text{ s}) = \frac{1}{3} i^{\text{max}}$, we find the inductive time constant from

$$i^{\text{max}} (1 - e^{-5.00 \text{ s}/\tau_L}) = \frac{1}{3} i^{\text{max}} \quad \Rightarrow \quad \tau_L = -5.00 \text{ s} / \ln \frac{2}{3} = 12.3 \text{ s}.$$

28. (a) Case I: When switch S is just closed (case I), $\Delta V_1 = \mathcal{E}$, so

$$i_1 = \mathcal{E}/R_1 = 10 \text{ V}/5.0\Omega = 2.0 \text{ A}.$$

Case II: After a long time, we still have $\Delta V_1 = \mathcal{E}$, so $i_1 = 2.0 \text{ A}$.

(b) Case I: Initially, $\Delta V_L = \mathcal{E}$, so $\Delta V_{R_2} = 0 \text{ V}$. Hence $i_2 = 0 \text{ A}$.

Case II: After a long time, $\Delta V_L = 0 \text{ V}$, so $\Delta V_{R_2} = \mathcal{E}$. Hence

$$i_2 = \mathcal{E}/R_2 = 10 \text{ V}/10\Omega = 1.0 \text{ A}.$$

(c) Case I: From the junction rule: $i = i_1 + i_2 = 2.0 \text{ A}$.

Case II: After a long time, $i = i_1 + i_2 = 3.0 \text{ A}$.

(d) Case I: Since $\Delta V_L = \mathcal{E}$, we have $\Delta V_{R2} = 0 \text{ V}$.

Case II: After a long time, $\Delta V_L = 0 \text{ V}$, so $\Delta V_{R2} = \mathcal{E} = 10 \text{ V}$.

(e) Case I: Initially, $\Delta V_L = \mathcal{E} = 10 \text{ V}$.

Case II: After a long time, $\Delta V_L = 0 \text{ V}$

(f) Case I: Initially, $\frac{di_2}{dt} = \frac{\mathcal{E}_L}{L} = \frac{10 \text{ V}}{5.0 \text{ H}} = 2.0 \text{ A/s}$.

Case II: After a long time, $\frac{di_2}{dt} = \frac{\mathcal{E}_L}{L} = 0 \text{ A/s}$.

31. (a) The current is given by Eq. 32-22:

$$i(t) = \frac{\mathcal{E}}{R} (1 - \exp(-Rt/L)).$$

With a resistance of $R = 1.00 \Omega$, we find the time required to reach a current of $i = 2.00 \text{ A}$ is

$$t = -\frac{L}{R} \ln \left(1 - \frac{iR}{\mathcal{E}} \right) = -\frac{18 \times 10^{-3} \text{ H}}{1.00 \Omega} \ln \left(1 - \frac{(2.00 \text{ A})(1.00 \Omega)}{12 \text{ V}} \right) = 3.28 \text{ ms}.$$

(b) With a resistance of $R = 5.00 \Omega$, the time required is

$$t = -\frac{L}{R} \ln \left(1 - \frac{iR}{\mathcal{E}} \right) = -\frac{18 \times 10^{-3} \text{ H}}{5.00 \Omega} \ln \left(1 - \frac{(2.00 \text{ A})(5.00 \Omega)}{12 \text{ V}} \right) = 6.45 \text{ ms}.$$

(c) With a resistance of $R = 6.00 \Omega$, the time required is

$$t = -\frac{L}{R} \ln \left(1 - \frac{iR}{\mathcal{E}} \right) = -\frac{18 \times 10^{-3} \text{ H}}{6.00 \Omega} \ln \left(1 - \frac{(2.00 \text{ A})(6.00 \Omega)}{12 \text{ V}} \right) = \infty.$$

(d) With a resistance of $R = 6.00 \Omega$, a current of $i = 2.00 \text{ A}$ is the steady state current that is asymptotically approached by the circuit. As such, it formally requires an infinite amount of time to reach that value.

(e) From the observed answers, it appears that the smaller the resistance, the smaller the time required to reach our target current. We expect, then, that $R = 0\Omega$ would lead to the shortest time.

(f) There are a number of approaches possible—one might take a graphical or numerical approach. Instead, using L'Hôpital's rule, we'll take the limit of the above function as $R \rightarrow 0\Omega$:

$$\begin{aligned}\lim_{R \rightarrow 0\Omega} t &= -L \lim_{R \rightarrow 0\Omega} \left[\frac{\ln(1 - iR/\mathcal{E})}{R} \right] = -L \lim_{R \rightarrow 0\Omega} \left[\frac{(-i/\mathcal{E})/(1 - iR/\mathcal{E})}{1} \right] \\ &= \frac{iL}{\mathcal{E}} = \frac{(2.00\text{A})(18 \times 10^{-3}\text{H})}{12\text{V}} = 3.00 \text{ ms}.\end{aligned}$$

57. The dip angle is the angle the magnetic field makes with the horizontal. The horizontal component of the field is therefore given by $B_x = B \cos \theta_{\text{dip}}$. Therefore, we can calculate the field from its horizontal component by

$$B = \frac{B_x}{\cos \theta_{\text{dip}}} = \frac{16\mu\text{T}}{\cos 73^\circ} = 55\mu\text{T}.$$