

## Questions

1. Explain in your own words “carrying capacity.”

**Answer:** The carrying capacity of a system is the amount of “users” the system can sustain indefinitely, as long as the system does not undergo any radical changes. For example, think of a mountain meadow that is grazed by wild goats. If the population of the goats gets too high (above the carrying capacity of the plants in the meadow), the flora will not be able to regenerate quickly enough to maintain a fairly constant amount of material. This is bad news for the plants, but also bad news for the goats.

2. Under what circumstances is population growth sustainable? What does it mean for something to be sustainable?

**Answer:** *No amount of growth in the natural world is sustainable indefinitely.* At some point, some factor will prevent the growth from continuing. With respect to populations, all living creatures require resources. Some are very tangible and immediate – clean water, nutrients, space. As a population grows, it will, at some point, exhaust some limiting resource and stop growing.

Something is sustainable when the rate at which it is used is at or below the rate at which it is replenished.

3. In Problem 9, I wrote that using “the equation of exponential change” will only get you “a better estimate.” Explain why equations describing natural phenomena sometimes only give estimates as opposed to “exact” answers.

**Answer:** The mathematical model that the equation of exponential change represents ( $N = N_0 e^{at}$ ) is a precise, smoothly varying function, with no regard for minor (or major) unforeseen events that could alter the growth pattern of the population. Also, the equation predicts non-integer numbers of individuals (22.45 fish, for example) at nearly every instant of time, which is clearly absurd. These issues do not render the equation useless. In fact, it’s quite powerful. In that short expression is tied up a very good estimate (given certain assumptions) of the population at any instant in time. In general, there are a lot of subtle aspects of natural phenomena that are simply too complex to try to explain using equations. As such, any mathematical model can only provide estimates of what is going to happen, or provide somewhat incomplete explanation of something that did happen. The best models are the ones that incorporate the most detail.

4. Detail a real-world example of natural resource use where the Tragedy of the Commons plays a realistically limiting role in the consumption of that resource.

**Answer:** The consumption of oil is a great example. No matter how friendly we are to each other, no matter how much planning or legislation we put into place, no matter how many clever ways we come up with to try to extract more and more oil from the planet, eventually we will ostensibly use up all of the oil. We use it as a resource at a rate that is much, much, much higher than the rate at which it is replenished. (We will have consumed nearly all of the world’s oil reserves in a matter of century and it takes tens of millions of years for it to be replenished.)

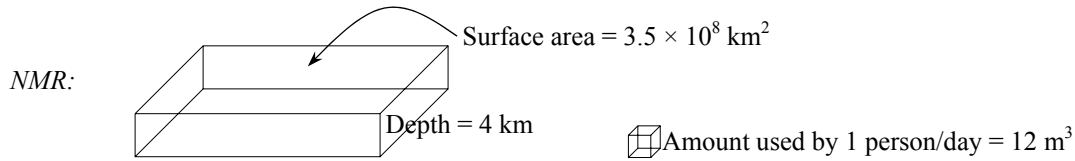
5. When scientists first determined that CFCs were going to negatively impact atmospheric ozone, some thought it would have the greatest detrimental effects in the tropics (more intense sunlight). Real data from the atmosphere revealed that the most deleterious effects were at the poles. Explain how this process reveals something about the *strengths* and potential *pitfalls* of science.

**Answer:** *Strengths* – (a) Science provides a framework in which we can make predictions about things that have not yet come to pass. (b) Science requires those predictions to be checked. If the predictions fail, the scientific framework forces people to go back and rethink how processes are related.

*Pitfalls* – (a) It is very easy to think that science can provide all of the answers or that scientifically-based predictions are infallible. (b) Even models that have predictive power always need to be revised and improved at some point as more data is collected and as understanding grows.

## Problems

6. The oceans of the Earth cover about  $3.5 \times 10^8$  km<sup>2</sup> and have an average depth of 4 km. What’s the volume of water on the planet? The US consumes about 12 m<sup>3</sup> per person of water every day to support all activities (this includes agriculture, industry, et cetera). At this rate, how many people could the Earth support if water (even salt water) were the limiting factor? Compare this number to some the author derives in Chapter 2, and suggest if this is a reasonable maximum number of people for the Earth to support.



**Knowns:** Surface area of the oceans =  $3.5 \times 10^8 \text{ km}^2$   
 Ocean depth = 4 km  
 Each person in the US uses about  $12 \text{ m}^3$  of water/day  
 $1 \text{ km} = 10^3 \text{ m}$

**Unknowns:** Volume of ocean water  
 How many people that will support

**Assumptions:** Ignore fresh water and only consider ocean water  
 Water that gets used is recycled back into the system every day  
 As stated, the rate of consumption is uniform for each person in the world

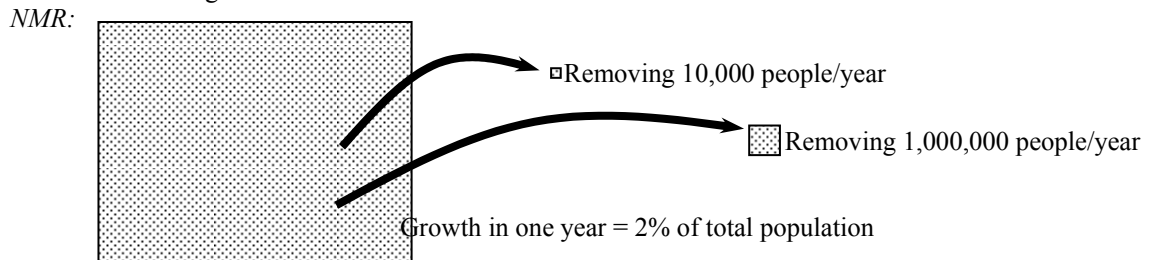
**SOP:** How much water is available is the ratio of the volumes of the ocean water to the amount each person uses

**Equations:** Volume = surface area  $\times$  depth; # of people = amount/(amount/person)  
**Solution:** The volume of water in the ocean is  $3.5 \times 10^8 \text{ km}^2 \times 4 \text{ km} = 14 \times 10^8 \text{ km}^3$ .  
 We need to convert this from  $\text{km}^3$  to  $\text{m}^3$ :  $(1 \text{ km})^3 = (10^3 \text{ m})^3 = 10^9 \text{ m}^3$ , so

$$\frac{14 \times 10^8 \cancel{\text{km}^3}}{1} \times \frac{10^9 \text{ m}^3}{\cancel{1 \text{ km}^3}} = 14 \times 10^{17} \text{ m}^3 \text{ is how much water we have available. If each person uses}$$

$12 \text{ m}^3/\text{day}$ , then we've got enough water for  $14 \times 10^{17} \text{ m}^3 / (12 \text{ m}^3/\text{person}) = 1.2 \times 10^{17}$  people. This is a ridiculously large number compared to some of the more realistic estimates made by the author. Clearly, if we could utilize the saltwater on Earth, then that, in and of itself, would not be a limiting factor to human population growth. Of course even consuming a significant fraction of the ocean's water would certainly lead to other problems.

7. Given the current growth rate of people on Earth, how much of an effect would moving 10,000 people off the planet to Mars, the Moon or wherever, have on human population growth? What if we moved 1 million people per year, would that have a big effect?



**Knowns:** We are going to remove 10,000 people/year and 1 million people/year  
 The growth rate is currently about 2%/year  
 There are about 6 billion people

**Unknowns:** The fraction of the number of individuals we add to this planet/year that would represent 10,000 or 1 million people/year

**Assumptions:** The growth rate doesn't change  
 We could actually move this many people in a year off the planet

**SOP:** The number of people we add each year is the product of the growth and the current number of people.

The fraction of that growth that we could move off the planet is given by the quotient of the number we can move off the planet and the number of people added per year

**Equations:** # of people gained/year = current population  $\times$  growth rate  
 fraction able to move = (number moved/year)/(# of people gained/year)  
**Solution:** # of people gained/year =  $6,000,000,000 \times 0.02 = 120,000,000$  people/year

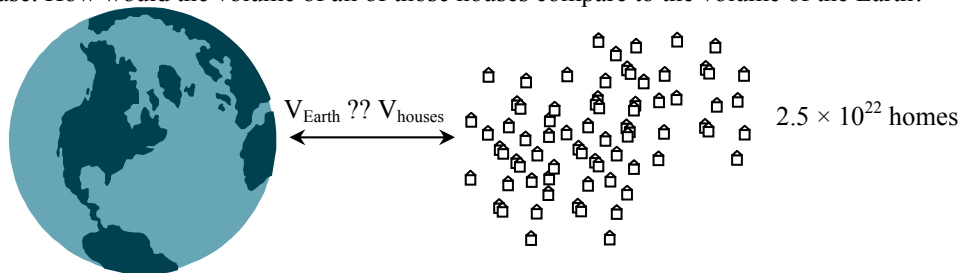
If we could move only 10,000 people/year, that would represent  
 $(10,000 \text{ people moved/year}) / (120,000,000 \text{ people gained/year}) = 0.008\%$ .

If we could move 1 million people/year, that would represent  
 $(1,000,000 \text{ people moved/year}) / (120,000,000 \text{ people gained/year}) = 0.8\%$ .

Neither of those cases has any significant impact on the growth of human population, so moving people off the planet is not a very realistic way to stop human population growth on Earth.

8. Let's say that the author's first estimate in Chapter 2 regarding the maximum number of people the Earth could hold ( $7.5 \times 10^{22}$  people) was somehow correct(!). And let's say the each of those people lived in a small house, 3 people to a house. How would the volume of all of those houses compare to the volume of the Earth?

NMR:



**Knowns:** Radius of the earth = 6371 km =  $6371 \times 10^3$  m

3 people to a house

$7.5 \times 10^{22}$  people

**Unknowns:** Volume of the Earth and the volume of all of the houses.

**Assumptions:** The earth is a perfect sphere (it is not)

Houses are simply box-shaped  $l = w = 10$  m, and  $h = 3$  m

**SOP:** Compare the volume of the Earth to the volume of all of the houses.

**Equations:**  $V_{\text{sphere}} = 4/3\pi r^3$ ;  $V_{\text{box}} = l \times w \times h$

**Solution:** Since we are putting 3 people/house, we have  $2.5 \times 10^{22}$  homes

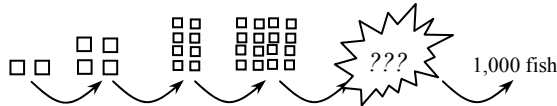
$$V_{\text{Earth}} = 4/3\pi r_E^3 = 4/3(3.14)(6371 \times 10^3 \text{ m})^3 = 1.1 \times 10^{21} \text{ m}^3$$

$$V_{\text{homes}} = (2.5 \times 10^{22} \text{ homes})(10 \text{ m} \times 10 \text{ m} \times 3 \text{ m}) = 7.5 \times 10^{24} \text{ m}^3$$

That means that the volume of all of the homes is  $7.5 \times 10^{24} \text{ m}^3 / 1.1 \times 10^{21} \text{ m}^3 = 6818$  times bigger than the Earth. Or, put another way, all of those homes would be big enough to make 6818 objects the same size as our planet! Clearly our population can not get that big.

9. A certain population of fish has a growth rate of 20.5% per year. If there are initially 2 individuals in the population, how long will it take for that population to reach 1,000 fish? Use two methods: the Magic Number to get a rough estimate of the time and the equation of exponential change to get a better estimate.

NMR:



**Knowns:** Start with 2 fish

**Unknowns:** How long it will take until we have 1,000 fish

**Assumptions:** Magic number is about 70

The fish population continues to grow at smooth rate of 20.5%/year.

**SOP:** The fish population increase exponentially with respect to time. The time for the population to double is directly related to the growth rate

**Equations:** Doubling time =  $70/\text{rate}$

$$N = N_o e^{\alpha t}$$

**Solution:** By the first method, the doubling time is  $70/(20.5\%/\text{year}) = 3.4$  years.

So, how many doublings does it take for the population to go from 2 to 1,000? Using my calculator I found that the population grew like this:  $2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 32$ , and so on until I reached 1024 after 9 doublings. So,  $9 \times 3.4$  years = 30.6 years. Since I overshot 1,000 by a little, let's say it took about 30 years.

Using the equation of exponential change, I need to solve for time,  $t$ .

$$N = N_o e^{\alpha t} \rightarrow \frac{N}{N_o} = e^{\alpha t} \rightarrow \ln\left(\frac{N}{N_o}\right) = \alpha t \rightarrow t = \frac{1}{\alpha} \ln\left(\frac{N}{N_o}\right)$$

For this population of fish,  $\alpha = 0.205$ . Plugging the numbers into the above expression yields:

$$t = \frac{1}{\alpha} \ln\left(\frac{N}{N_o}\right) = \frac{1}{0.205} \ln\left(\frac{1,000}{2}\right) = 30.3 \text{ years.}$$

The two answers are a pretty close match – great!