Technical Appendix to "Can Non-traded Goods Solve the 'Comovement Problem?"'

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1. Planner's Problem (Benchmark Version)

The equilibrium of the economy detailed in Section 5 of the paper consists of a set of functions defining the behavior of consumption, investment, output, etc., as functions of the model's exogenous shocks and capital stocks. For calculational ease, some additional variables are introduced. These include lagged labor services, $LN_{t+1}^S = N_t^S$ for each sector. To ease working with the endogenous discount factor setup, define an auxiliary variable, Ξ, as follows:

$$\Xi_t = E_t \sum_{j=t+1}^{\infty} \left(\frac{\Delta_j}{\Delta_t}\right) \frac{1}{1-\sigma} \left\{ \left[\left(c_t^{TR}\right)^{-\psi} + \left(c_t^{NT}\right)^{-\psi} \right]^{-\frac{1}{\psi}} \right\}^{1-\sigma} L_t^{\chi}.$$

The law of motion for Ξ is given by:

$$\begin{aligned} E_t\left(\Xi_{t+1}\right) &= \left[1 + \left[\left(c_t^{TR}\right)^{-\psi} + \left(c_t^{NT}\right)^{-\psi}\right]^{-\frac{1}{\psi}} L_t^{\frac{\chi}{1-\sigma}}\right]^{\beta} \Xi_t \\ &\quad + E_t\left[\frac{1}{1-\sigma}\left\{\left[\left(c_{t+1}^{TR}\right)^{-\psi} + \left(c_{t+1}^{NT}\right)^{-\psi}\right]^{-\frac{1}{\psi}}\right\}^{1-\sigma} L_{t+1}^{\chi}\right] \end{aligned}$$

An analogous version of Ξ exists for the foreign country. Finally, for notational brevity, I have replaced the traded and non-traded sectoral production functions with f and g, respectively and have introduced the following functional simplification:

$$\Upsilon_t = \beta \ln \left\{ 1 + \left[\left(c_{t+1}^{TR} \right)^{-\psi} + \left(c_{t+1}^{NT} \right)^{-\psi} \right]^{-\frac{1}{\psi}} L_t^{\frac{\chi}{1-\sigma}} \right\},$$

where an analogous version of Υ exists for the foreign country.

The model is solved by considering the problem faced by a social planner who chooses a sequence $\{c_t^{TR}, c_t^{NT}, L_t, N_t^{TR}, N_t^{NT}, c_t^{TR*}, c_t^{NT*}, L_t^*, \omega_t, \omega_t^*, N_t^{S,TR}, N_t^{S,NT}, N_t^{S,TR*}, N_t^{S,NT*}, LN_{t+1}^{S,TR}, LN_{t+1}^{S,NT}, LN_{t+1}^{S,TR*}, LN_{t+1}^{S,NT*}, i_t^{TR}, i_t^{NT}, i_t^{TR*}, i_t^{NT*}, k_{t+1}^{TR}, k_{t+1}^{NT}, k_{t+1}^{TR*}, k_{t+1}^{NT*}, B_{t+1}, s_{2t}, s_{4t}, z_{2t}, z_{4t}, \Theta_t^{TR}, \Theta_t^{NT}, \Theta_t^{TR*}, \Theta_t^{NT*}, \Phi_t^{TR}, \Phi_t^{NT}, \Phi_t^{TR*}, \Phi_t^{NT*}, \lambda_t^{TR}, \lambda_t^{NT}, \lambda_t^{TR*}, \lambda_t^{NT*}, \Psi_t^B, \Psi_t^{NT}, \Psi_t^{NT*}, \Psi_t^W, \Gamma_t^{s_2}, \Gamma_t^{s_4}, \Gamma_t^{z_2}, \Gamma_t^{z_4}\}$ to maximize:

$$\begin{aligned}
\Im \quad = \quad & \pi \left\{ E_0 \sum_{t=0}^{\infty} \Delta_t u \left(c_t^{TR}, c_t^{NT}, L_t \right) \right. \\
& + \omega_t \left(1 - L_t - N_t^{TR} - N_t^{NT} \right) \\
& + \Theta_t^{TR} \left(LN_{t+1}^{S,TR} - \zeta \left(\frac{N_t^{TR}}{LN_t^{S,TR}} \right) LN_t^{S,TR} \right) \\
& + \Theta_t^{NT} \left(LN_{t+1}^{S,NT} - \zeta \left(\frac{N_t^{NT}}{LN_t^{S,NT}} \right) LN_t^{S,NT} \right) \\
& + \Phi_t^{TR} \left(LN_{t+1}^{S,TR} - N_t^{S,TR} \right) \\
& + \Phi_t^{NT} \left(LN_{t+1}^{S,NT} - N_t^{S,NT} \right) \\
& + \lambda_t^{TR} \left(\gamma k_{t+1}^{TR} - (1-\delta) k_t^{TR} - \phi \left(\frac{i_t^{TR}}{k_t^{TR}} \right) k_t^{TR} \right) \\
& + \lambda_t^{NT} \left(\gamma k_{t+1}^{NT} - (1-\delta) k_t^{NT} - \phi \left(\frac{i_t^{NT}}{k_t^{NT}} \right) k_t^{NT} \right) \\
& + \Psi_t^B \left(\begin{array}{c} a_t^{TR} f \left(k_t^{TR}, N_t^{S,TR} \right) + a_t^{NT} g \left(k_t^{NT}, N_t^{S,NT} \right) / p_t^{NT} - \\ \left(c_t^{TR} + i_t^{TR} \right) / p_t^{s_1} - \left(c_t^{NT} + i_t^{NT} \right) / p_t^{s_2} + (1 + R_t) B_t - B_{t+1} \end{array} \right) \\
& + \Psi_t^{NT} \left(a_t^{NT} g \left(k_t^{NT}, N_t^{S,NT} \right) - \left(c_t^{NT} + i_t^{NT} \right) / p_t^{s_5} + \left[p_t^{NT} \left(s_{2t} + q_t z_{4t} \right) \right] \right) \\
& + \Gamma_t^{s_2} \left(p_t^{s_2} s_{2t} \left[1 + q_t^{1-\rho} \left(\frac{\omega_3}{\omega_2} \right)^{-\rho} + \left(p_t^{NT} \right)^{-(1+\rho)} \left(\frac{\omega_4}{\omega_3} \right)^{-\rho} \right] - \left(c_t^{NT} + i_t^{NT} \right) \right) \\
& \left. + \Gamma_t^{z_4} \left(z_{4t} - q_t^{-\rho} \left(\frac{\omega_3}{\omega_5} \right)^{-\rho} s_{2t} \right) \right\} \\
& + (1-\pi) \left\{ E_0 \sum_{t=0}^{\infty} \Delta_t^* u \left(c_t^{TR*}, c_t^{NT*}, L_t^* \right) \right. \\
& + \omega_t^* \left(1 - L_t^* - N_t^{TR*} - N_t^{NT*} \right)
\end{aligned}$$

$$
\begin{aligned}
&+\Theta_t^{TR*}\left(LN_{t+1}^{S,TR*}-\zeta\left(\frac{N_t^{TR*}}{LN_t^{S,TR*}}\right)LN_t^{S,TR*}\right)\\
&+\Theta_t^{NT*}\left(LN_{t+1}^{S,NT*}-\zeta\left(\frac{N_t^{NT*}}{LN_t^{S,NT*}}\right)LN_t^{S,NT*}\right)\\
&+\Phi_t^{TR*}\left(LN_{t+1}^{S,TR*}-N_t^{S,TR*}\right)\\
&+\Phi_t^{NT*}\left(LN_{t+1}^{S,NT*}-N_t^{S,NT*}\right)\\
&+\lambda_t^{TR*}\left(\gamma k_{t+1}^{TR*}-(1-\delta)k_t^{TR*}-\phi\left(\frac{i_t^{TR*}}{k_t^{TR*}}\right)k_t^{TR*}\right)\\
&+\lambda_t^{NT*}\left(\gamma k_{t+1}^{NT*}-(1-\delta)k_t^{NT*}-\phi\left(\frac{i_t^{NT*}}{k_t^{NT*}}\right)k_t^{NT*}\right)\\
&+\Psi_t^{NT*}\left(a_t^{NT*}g\left(k_t^{NT*},N_t^{S,NT*}\right)-\left(c_t^{NT*}+i_t^{NT*}\right)/p_t^{z_5}+\left[p_t^{NT*}\left(z_{2t}+\frac{1}{q_t}s_{4t}\right)\right]\right)\\
&+\Psi_t^{W}\left(a_t^{TR*}f\left(k_t^{TR*},N_t^{S,TR*}\right)-\left(c_t^{TR*}+i_t^{TR*}\right)/p_t^{s_3}-\left[z_{2t}+z_{3t}+z_{4t}-\frac{1}{q_t}s_{3t}\right]\right)\\
&+\Gamma_t^{z_2}\left(p_t^{z_2}z_{2t}\left[q_t^{\rho-1}\left(\frac{\omega_4}{\omega_5}\right)^{-\rho}+\left(p_t^{NT*}\right)^{\rho-1}\left(\frac{\omega_4}{\omega_3}\right)^{-\rho}+1\right]-\left(c_t^{NT*}+i_t^{NT*}\right)\right)\\
&+\Gamma_t^{s_4}\left(s_{4t}-q_t^{\rho}\left(\frac{\omega_3}{\omega_5}\right)^{-\rho}z_{2t}\right)\Bigg\}.
\end{aligned}
$$

$\Theta_t^{TR},\Theta_t^{NT},\Theta_t^{TR*},\Theta_t^{NT*},\Phi_t^{TR},\Phi_t^{NT},\Phi_t^{TR*},\Phi_t^{NT*},\lambda_t^{TR},\lambda_t^{NT},\lambda_t^{TR*},\lambda_t^{NT*},\Psi_t^{B},\Psi_t^{NT},\Psi_t^{NT*},\Psi_t^{W},\Gamma_t^{s_2}$, $\Gamma_t^{z_4},\Gamma_t^{s_4},\Gamma_t^{z_2}$ are Lagrange multipliers, and are interpreted as utility-denominated shadow prices; Θ^{TR}, Θ^{NT}, Θ^{TR*} and Θ^{NT*} are the marginal products of labor services; Φ^{TR}, Φ^{NT}, Φ^{TR*} and Φ^{NT*} are the marginal products of labor; λ^{TR}, λ^{NT}, λ^{TR*}, λ^{NT*}, are the marginal products of capital; Ψ^B is the price of domestic tradeables, Ψ^{NT} and Ψ^{NT*} the price of domestic and foreign non-tradeables, respectively, and Ψ^W is the price of foreign tradeables; and the Γs are the composite shadow prices of the domestic and foreign consumption and investment goods. The first-order conditions for an interior solution are:

$$\left(c_t^{TR}\right): \quad D_1u\left(c_t^{TR},c_t^{NT},L_t\right)-\Psi_t^B/p_t^{s_1}-\Xi_tD_1\Upsilon\left(c_t^{TR},c_t^{NT},L_t\right)=0 \tag{A1}$$

$$\left(c_t^{NT}\right): \quad D_2u\left(c_t^{TR},c_t^{NT},L_t\right)-\Psi_t^{NT}/p_t^{s_5}-\Gamma_t^{s_2}-\Xi_tD_2\Upsilon\left(c_t^{TR},c_t^{NT},L_t\right)=0 \tag{A2}$$

$$(L_t): \quad D_3u\left(c_t^{TR},c_t^{NT},L_t\right)-\omega_t-\Xi_tD_3\Upsilon\left(c_t^{TR},c_t^{NT},L_t\right)=0 \tag{A3}$$

$$\left(N_t^{TR}\right): \quad -\omega_t-\Theta_t^{TR}\zeta'\left(\frac{N_t^{TR}}{LN_t^{S,TR}}\right)=0 \tag{A4}$$

$$\left(N_t^{NT}\right): \quad -\omega_t-\Theta_t^{NT}\zeta'\left(\frac{N_t^{NT}}{LN_t^{S,NT}}\right)=0 \tag{A5}$$

$$\left(N_t^{S,TR}\right): \quad \Psi_t^Ba_t^{TR}D_2f\left(k_{t+1}^{TR},N_{t+1}^{S,TR}\right)-\Phi_t^{TR}=0 \tag{A6}$$

$$\left(N_t^{S,NT}\right): \quad \Psi_t^{NT} a_t^{NT} D_2 g\left(k_{t+1}^{NT}, N_{t+1}^{S,NT}\right) - \Phi_t^{NT} = 0 \tag{A7}$$

$$\left(LN_{t+1}^{S,TR}\right): \quad \begin{array}{c} \Delta_t\left(\Theta_t^{TR} + \Phi_t^{TR}\right) - \\ \Delta_{t+1} E\left\{\Theta_{t+1}^{TR}\left[\zeta\left(\frac{N_{t+1}^{TR}}{LN_{t+1}^{S,TR}}\right) - \frac{N_{t+1}^{TR}}{LN_{t+1}^{S,TR}}\zeta'\left(\frac{N_{t+1}^{TR}}{LN_{t+1}^{S,TR}}\right)\right]\right\} = 0 \end{array} \tag{A8}$$

$$\left(LN_{t+1}^{S,NT}\right): \quad \begin{array}{c} \Delta_t\left(\Theta_t^{NT} + \Phi_t^{NT}\right) - \\ \Delta_{t+1} E\left\{\Theta_{t+1}^{NT}\left[\zeta\left(\frac{N_{t+1}^{NT}}{LN_{t+1}^{S,NT}}\right) - \frac{N_{t+1}^{NT}}{LN_{t+1}^{S,NT}}\zeta'\left(\frac{N_{t+1}^{NT}}{LN_{t+1}^{S,NT}}\right)\right]\right\} = 0 \end{array} \tag{A9}$$

$$(\omega_t): \quad 1 - L_t - N_t^{TR} - N_t^{NT} = 0 \tag{A10}$$

$$\left(i_t^{TR}\right): \quad -\Psi_t^B / p_t^{s_1} - \lambda_t^{TR} \phi'\left(i_t^{TR}/k_t^{TR}\right) = 0 \tag{A11}$$

$$\left(i_t^{NT}\right): \quad -\Psi_t^{NT}/p_t^{s_5} - \Gamma_t^{s_2} - \lambda_t^{NT}\phi'\left(i_t^{NT}/k_t^{NT}\right) = 0 \tag{A12}$$

$$\left(k_{t+1}^{TR}\right): \quad \begin{array}{c} \gamma\Delta_t\lambda_t^{TR} + \Delta_{t+1}E\left\{\Psi_{t+1}^B a_{t+1}^{TR} D_1 f\left(k_{t+1}^{TR}, N_{t+1}^{S,TR}\right) - \right. \\ \left. \lambda_{t+1}^{TR}\left[(1-\delta) - \phi\left(i_{t+1}^{TR}/k_{t+1}^{TR}\right) + \left(i_{t+1}^{TR}/k_{t+1}^{TR}\right)\phi'\left(i_{t+1}^{TR}/k_{t+1}^{TR}\right)\right]\right\} = 0 \end{array} \tag{A13}$$

$$\left(k_{t+1}^{NT}\right): \quad \begin{array}{c} \gamma\Delta_t\lambda_t^{NT} + \Delta_{t+1}E\left\{\Psi_{t+1}^{NT} a_{t+1}^{TR} D_1 g\left(k_{t+1}^{NT}, N_{t+1}^{S,NT}\right) - \right. \\ \left. \lambda_{t+1}^{NT}\left[(1-\delta) - \phi\left(i_{t+1}^{NT}/k_{t+1}^{NT}\right) + \left(i_{t+1}^{NT}/k_{t+1}^{NT}\right)\phi'\left(i_{t+1}^{NT}/k_{t+1}^{NT}\right)\right]\right\} = 0 \end{array} \tag{A14}$$

$$(B_{t+1}): \quad \Delta_t\Psi_t^B - \Delta_{t+1}\Psi_{t+1}^B\left(1 + R_{t+1}\right) = 0 \tag{A15}$$

$$(s_{2t}): \quad \Psi_t^{NT} p_t^{NT} + \Gamma_t^{s_2} p_t^{s_2}\left[1 + q_t^{1-\rho}\left(\frac{\omega_3}{\omega_2}\right)^{-\rho} + \left(p_t^{NT}\right)^{-(1-\rho)}\left(\frac{\omega_4}{\omega_3}\right)^{-\rho}\right] - \Gamma_t^{z_4} q_t^{-\rho}\left(\frac{\omega_3}{\omega_5}\right)^{-\rho} = 0 \tag{A16}$$

$$(s_{4t}): \quad \Psi_t^{NT*} p_t^{NT*}/q_t + \Gamma_t^{s_4} = 0 \tag{A17}$$

$$(z_{2t}): \quad \Psi_t^{NT*} p_t^{NT*} + \Gamma_t^{z_2} p_t^{z_2}\left[q_t^{\rho-1}\left(\frac{\omega_3}{\omega_5}\right)^{-\rho} + \left(p_t^{NT*}\right)^{\rho-1}\left(\frac{\omega_4}{\omega_3}\right)^{-\rho} + 1\right] - \Gamma_t^{s_4} q_t^{\rho}\left(\frac{\omega_2}{\omega_5}\right)^{-\rho} = 0 \tag{A18}$$

$$(z_{4t}): \quad \pi\Psi_t^{NT} p_t^{NT} q_t + \Gamma_t^{z_4} = 0 \tag{A19}$$

$$\left(c_t^{TR*}\right): \quad D_1 u\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) - \Psi_t^W / p_t^{s_3} - \Xi_t^* D_1 \Upsilon_t^*\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) = 0 \tag{A20}$$

$$(d_t^*): \quad D_2 u\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) - \Psi_t^{NT*}/p_t^{z_5} - \Gamma_t^{z_2} - \Xi_t^* D_2\Upsilon_t^*\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) = 0 \tag{A21}$$

$$(L_t^*): \quad D_3 u\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) - \omega_t - \Xi_t^* D_3\Upsilon_t^*\left(c_t^{TR*}, c_t^{NT*}, L_t^*\right) = 0 \tag{A22}$$

$$\left(N_t^{TR*}\right): \quad -\omega_t^* - \Theta_t^{TR*}\zeta'\left(\frac{N_t^{TR*}}{LN_t^{S,TR*}}\right) = 0 \tag{A23}$$

$$\left(N_t^{NT*}\right): \quad -\omega_t^* - \Theta_t^{NT*}\zeta'\left(\frac{N_t^{NT*}}{LN_t^{S,NT*}}\right) = 0 \tag{A24}$$

$$\left(N_t^{S,TR*}\right): \quad \Psi_t^W q_t a_t^{TR*} D_2 f\left(k_{t+1}^{TR*}, N_{t+1}^{S,TR*}\right) - \Phi_t^{TR*} = 0 \tag{A25}$$

$$\left(N_t^{S,NT*}\right): \quad \Psi_t^{NT*} a_t^{NT*} D_2 g\left(k_{t+1}^{NT*}, N_{t+1}^{S,NT*}\right) - \Phi_t^{NT*} = 0 \tag{A26}$$

$$\left(LN_{t+1}^{S,TR*}\right): \quad \begin{gathered}\Delta_t^*\left(\Theta_t^{TR*}+\Phi_t^{TR*}\right)- \\ \Delta_{t+1}^* E\left\{\Theta_{t+1}^{TR*}\left[\zeta\left(\frac{N_{t+1}^{TR*}}{LN_{t+1}^{S,TR*}}\right)-\frac{N_{t+1}^{TR*}}{LN_{t+1}^{S,TR*}}\zeta'\left(\frac{N_{t+1}^{TR*}}{LN_{t+1}^{S,TR*}}\right)\right]\right\}=0\end{gathered} \tag{A27}$$

$$\left(LN_{t+1}^{S,NT*}\right): \quad \begin{gathered}\Delta_t^*\left(\Theta_t^{NT*}+\Phi_t^{NT*}\right)- \\ \Delta_{t+1}^* E\left\{\Theta_{t+1}^{NT*}\left[\zeta\left(\frac{N_{t+1}^{NT*}}{LN_{t+1}^{S,NT*}}\right)-\frac{N_{t+1}^{NT*}}{LN_{t+1}^{S,NT*}}\zeta'\left(\frac{N_{t+1}^{NT*}}{LN_{t+1}^{S,NT*}}\right)\right]\right\}=0\end{gathered} \tag{A28}$$

$$(\omega_t^*): \quad 1-L_t^*-N_t^{TR*}-N_t^{NT*}=0 \tag{A29}$$

$$\left(i_t^{TR*}\right): \quad -\Psi_t^W/p_t^{s_3}-\lambda_t^{TR*}\phi'\left(i_t^{TR*}/k_t^{TR*}\right)=0 \tag{A30}$$

$$\left(i_t^{NT*}\right): \quad -\Psi_t^{NT}/p_t^{z_5}-\Gamma_t^{z_2}-\lambda_t^{NT*}\phi'\left(i_t^{NT*}/k_t^{NT*}\right)=0 \tag{A31}$$

$$\left(k_{t+1}^{TR*}\right): \quad \begin{gathered}\gamma\Delta_t^*\lambda_t^{TR*}+\Delta_{t+1}^* E\left\{\Psi_{t+1}^W a_{t+1}^{TR*} D_1 f\left(k_{t+1}^{TR*}, N_{t+1}^{S,TR*}\right)- \right.\\ \left.\lambda_{t+1}^{TR*}\left[(1-\delta)-\phi\left(i_{t+1}^{TR*}/k_{t+1}^{TR*}\right)+\left(i_{t+1}^{TR*}/k_{t+1}^{TR*}\right)\phi'\left(i_{t+1}^{TR*}/k_{t+1}^{TR*}\right)\right]\right\}=0\end{gathered} \tag{A32}$$

$$\left(k_{t+1}^{NT*}\right): \quad \begin{gathered}\gamma\Delta_t^*\lambda_t^{NT*}+\Delta_{t+1}^* E\left\{\Psi_{t+1}^{NT*} a_{t+1}^{NT*} D_1 g\left(k_{t+1}^{NT*}, N_{t+1}^{S,NT*}\right)- \right.\\ \left.\lambda_{t+1}^{NT*}\left[(1-\delta)-\phi\left(i_{t+1}^{NT*}/k_{t+1}^{NT*}\right)+\left(i_{t+1}^{NT*}/k_{t+1}^{NT*}\right)\phi'\left(i_{t+1}^{NT*}/k_{t+1}^{NT*}\right)\right]\right\}=0\end{gathered} \tag{A33}$$

$$\left(\Theta_t^{TR}\right): \quad LN_{t+1}^{S,TR}-\zeta\left(\frac{N_t^{TR}}{LN_t^{S,TR}}\right)LN_t^{S,TR}=0 \tag{A34}$$

$$\left(\Theta_t^{NT}\right): \quad LN_{t+1}^{S,NT}-\zeta\left(\frac{N_t^{NT}}{LN_t^{S,NT}}\right)LN_t^{S,NT}=0 \tag{A35}$$

$$\left(\Theta_t^{TR*}\right): \quad LN_{t+1}^{S,TR*}-\zeta\left(\frac{N_t^{TR*}}{LN_t^{S,TR*}}\right)LN_t^{S,TR*}=0 \tag{A36}$$

$$\left(\Theta_t^{NT*}\right): \quad LN_{t+1}^{S,NT*}-\zeta\left(\frac{N_t^{NT*}}{LN_t^{S,NT*}}\right)LN_t^{S,NT*}=0 \tag{A37}$$

$$\left(\Phi_t^{TR}\right): \quad LN_{t+1}^{S,TR}-N_t^{S,TR}=0 \tag{A38}$$

$$\left(\Phi_t^{NT}\right): \quad LN_{t+1}^{S,NT}-N_t^{S,NT}=0 \tag{A39}$$

$$\left(\Phi_t^{TR*}\right): \quad LN_{t+1}^{S,TR*}-N_t^{S,TR*}=0 \tag{A40}$$

$$\left(\Phi_t^{NT*}\right): \quad LN_{t+1}^{S,NT*}-N_t^{S,NT*}=0 \tag{A41}$$

$$\left(\lambda_t^{TR}\right): \quad \gamma k_{t+1}^{TR}-(1-\delta)k_t^{TR}-\phi\left(i_t^{TR}/k_t^{TR}\right)k_t^{TR}=0 \tag{A42}$$

$$\left(\lambda_t^{NT}\right): \quad \gamma k_{t+1}^{NT}-(1-\delta)k_t^{NT}-\phi\left(i_t^{NT}/k_t^{NT}\right)k_t^{NT}=0 \tag{A43}$$

$$\left(\lambda_t^{TR*}\right): \quad \gamma k_{t+1}^{TR*}-(1-\delta)k_t^{TR*}-\phi\left(i_t^{TR*}/k_t^{TR*}\right)k_t^{TR*}=0 \tag{A44}$$

$$\left(\lambda_t^{NT*}\right): \quad \gamma k_{t+1}^{NT*}-(1-\delta)k_t^{NT*}-\phi\left(i_t^{NT*}/k_t^{NT*}\right)k_t^{NT*}=0 \tag{A45}$$

$$\left(\Psi_t^B\right): \quad \begin{gathered}B_{t+1}-a_t^{TR}\left(k_t^{TR}\right)^{1-\alpha}\left(N_t^{S,TR}\right)^{\alpha}-a_t^{NT}\left(k_t^{NT}\right)^{1-\xi}\left(N_t^{S,NT}\right)^{\xi}/p_t^{NT}+ \\ \left(c_t^{TR}+i_t^{TR}\right)/p_t^{s_1}+\left(c_t^{NT}+i_t^{NT}\right)/p_t^{s_2}-(1+R_t)B_t=0\end{gathered} \tag{A46}$$

$$\left(\Psi_t^{NT}\right): \quad a_t^{NT}\left(k_t^{NT}\right)^{1-\xi}\left(N_t^{S,NT}\right)^{\xi}-\left(c_t^{NT}+i_t^{NT}\right)/p_t^{s_5}+\left[p_t^{NT}\left(s_{2t}+q_t z_{4t}\right)\right]=0 \tag{A47}$$

$$\left(\Psi_t^{NT*}\right): \quad a_t^{NT*}\left(k_t^{NT*}\right)^{1-\xi}\left(N_t^{S,NT*}\right)^{\xi}-\left(d_t^{*}+i_t^{NT*}\right)/p_t^{z_5}+\left[p_t^{NT*}\left(z_{2t}+\frac{1}{q_t}s_{4t}\right)\right]=0 \tag{A48}$$

$$\left(\Psi_t^{W}\right): \quad \begin{array}{c} \pi a_t^{TR}\left(k_t^{TR}\right)^{1-\alpha}\left(N_t^{S,TR}\right)^{\alpha}+(1-\pi)\,q_t a_t^{TR*}\left(k_t^{TR*}\right)^{1-\alpha}\left(N_t^{S,TR*}\right)^{\alpha}+ \\ \pi a_t^{NT}\left(k_t^{NT}\right)^{1-\xi}\left(N_t^{S,NT}\right)^{\xi}/p_t^{NT}+(1-\pi)\,q_t a_t^{NT*}\left(k_t^{NT*}\right)^{1-\xi}\left(N_t^{S,NT*}\right)^{\xi}/p_t^{NT*}- \\ \pi\left(c_t+i_t^{TR}\right)/p_t^{s_1}-\pi\left(d_t+i_t^{NT}\right)/p_t^{s_2}- \\ (1-\pi)\left(c_t^{*}+i_t^{TR*}\right)/p_t^{s_3}-(1-\pi)\left(d_t^{*}+i_t^{NT*}\right)/p_t^{s_4}=0 \end{array} \tag{A49}$$

$$\left(\Gamma_t^{s_2}\right): \quad p_t^{s_2}s_{2t}\left[1+q_t^{1-\rho}\left(\frac{\omega_3}{\omega_2}\right)^{-\rho}+\left(p_t^{NT}\right)^{-(1-\rho)}\left(\frac{\omega_4}{\omega_3}\right)^{-\rho}\right]-\left(c_t^{NT}+i_t^{NT}\right)=0 \tag{A50}$$

$$\left(\Gamma_t^{s_4}\right): \quad s_{4t}-q_t^{\rho}\left(\frac{\omega_3}{\omega_5}\right)^{-\rho}z_{2t}=0 \tag{A51}$$

$$\left(\Gamma_t^{z_2}\right): \quad p_t^{z_2}z_{2t}\left[\left(\frac{p_t^{NT*}}{q_t}\right)^{1-\rho}\left(\frac{\omega_4}{\omega_5}\right)^{-\rho}+\left(p_t^{NT*}\right)^{1-\rho}\left(\frac{\omega_4}{\omega_3}\right)^{-\rho}+1\right]-\left(c_t^{NT*}+i_t^{NT*}\right)=0 \tag{A52}$$

$$\left(\Gamma_t^{z_4}\right): \quad z_{4t}-q_t^{-\rho}\left(\frac{\omega_3}{\omega_5}\right)^{-\rho}s_{2t}=0 \tag{A53}$$

In the above expressions, D_j denotes the partial derivative of the function in precedes with respect to that function's jth argument.

To complete the description of the model's solution, four additional steps are necessary. First, the four resource constraints are combined into a world resource contraint (A49) after first order conditions are taken. This allows one to eliminate variables z_3 and s_3 outright and any Ψ^W terms in the first-order conditions for z_2 and z_4. Second, the evolution of the domestic discount factor is given by:

$$\frac{\Delta_{t+1}}{\Delta_t}=\left[1+\left[\left(c_t^{TR}\right)^{-\psi}+\left(c_t^{NT}\right)^{-\psi}\right]^{-\frac{1}{\psi}}L_t^{\frac{\chi}{1-\sigma}}\right]^{\beta}.$$

The foriegn discount factor evolves identically, differing only by the usual notation. Third, one needs to include the two equations defining the law of motion for the auxiliary variables Ξ and Ξ^*. Finally, the terms of trade and relative prices of non-tradeables are defined using a series of "flow" equations the make use of the shadow prices discussed above. Specifically,

$$q_t=\Gamma_t^{s_4}/\Gamma_t^{z_4}.$$

(Note that from 13(a) and 13(c), s_4 and z_4 have the same price as domestic and foreign traded goods, respectively. Thus, the shadow prices of s_4 and z_4 can be used to define the terms of trade.) The relative prices of non-tradeables are given by:

$$\begin{aligned} p_t^{NT} &= \Psi_t^{NT}/\Psi_t^{B}, \\ p_t^{NT*} &= \Psi_t^{NT*}/\Psi_t^{W}. \end{aligned}$$

Composite prices are determined again using the shadow prices described above and the aggregation procedure described in Section 4.3 of the text. It follows that:

$$
\begin{aligned}
\Psi_t^B p_t^{s_2} &= \Gamma_t^{s_2}, \\
\Gamma_t^{s_4} p_t^{s_4} &= \Gamma_t^{z_2}, \\
\Gamma_t^{z_4} p_t^{z_4} &= \Gamma_t^{s_2}, \\
\Psi_t^B p_t^{z_2} &= \Gamma_t^{z_2}, \\
p_t^{s_5} p_t^{NT} &= p_t^{s_2}, \\
p_t^{z_5} p_t^{NT*} &= p_t^{z_2}, \\
\Psi_t^B p_t^{s3} &= q_t.
\end{aligned}
$$

This completes the description of the equations describing the model's solution.

The model's solution proceeds in two steps. First, the set of efficiency conditions, constraints, identities, and flow equations describing this economy is linearized by taking a first-order Taylor series approximation around the model's steady state. This yields a system in which the variables are expressed as percentage deviations from steady state. That is, $\widehat{m}_t = (m_t - \overline{m}) / \overline{m}$, where $\overline{m}$ is the steady state value of m. For small percentage deviations, $\widehat{m}_t \simeq \ln(m_t) - \ln(\overline{m})$, the data generated by the model can be compared to actual logged data. The resulting linear system was then solved using the King and Watson (1995; 1998) algorithms.

2. Planner's Problem for "No Capital Trade" Model Variant

The equilibrium of the economy detailed in Section x is obtained by solving the problem faced by a social planner who chooses a sequence $\{c_{ht}, c_{ft}, c_t^{TR}, c_t^{NT}, L_t, N_t^{TR}, N_t^{NT}, c_{ht}^*, c_{ft}^*, c_t^{TR*}, c_t^{NT*}, L_t^*, \omega_t,$ $\omega_t^*, N_t^{S,TR}, N_t^{S,NT}, N_t^{S,TR*}, N_t^{S,NT*}, LN_t^{S,TR}, LN_t^{S,NT}, LN_t^{S,TR*}, LN_t^{S,NT*}, i_t^{TR}, i_t^{NT}, i_t^{TR*}, i_t^{NT*}, k_t^{TR},$ $k_t^{NT}, k_t^{TR*}, k_t^{NT*}, \;_t, \;_t^*, B_t, \Theta_t^{TR}, \Theta_t^{NT}, \Theta_t^{TR*}, \Theta_t^{NT*}, \Phi_t^{TR}, \Phi_t^{NT}, \Phi_t^{TR*}, \Phi_t^{NT*}, \lambda_t^{TR}, \lambda_t^{NT}, \lambda_t^{TR*}, \lambda_t^{NT*},$ $\Psi_t^B, \Psi_t^{NT}, \Psi_t^{NT*}, \Psi_t^W\}$ to maximize:

$$
\begin{aligned}
\Im &= \pi \left\{ E_0 \sum_{t=0}^{\infty} \Delta_t \pi u \left(c_t^{TR}, c_t^{NT}, L_t \right) + \right. \\
&+ \;_t \left[c_t - \left(\omega_1 c_{ht}^{-\mu} + \omega_2 c_{ft}^{-\mu} \right)^{-\frac{1}{\mu}} \right] \\
&+ \omega_t \left(1 - L_t - N_t^{TR} - N_t^{NT} \right) \\
&+ \Theta_t^{TR} \left(LN_{t+1}^{S,TR} - \zeta \left(\frac{N_t^{TR}}{LN_t^{S,TR}} \right) LN_t^{S,TR} \right) \\
&+ \Theta_t^{NT} \left(LN_{t+1}^{S,NT} - \zeta \left(\frac{N_t^{NT}}{LN_t^{S,NT}} \right) LN_t^{S,NT} \right) \\
&+ \Phi_t^{TR} \left(LN_{t+1}^{S,TR} - N_t^{S,TR} \right)
\end{aligned}
$$

$$
\begin{aligned}
&+\Phi_t^{NT}\left(LN_{t+1}^{S,NT}-N_t^{S,NT}\right)\\
&+\lambda_t^{TR}\left(\gamma k_{t+1}^{TR}-(1-\delta)k_t^{TR}-\phi\left(\frac{i_t^{TR}}{k_t^{TR}}\right)k_t^{TR}\right)\\
&+\lambda_t^{NT}\left(\gamma k_{t+1}^{NT}-(1-\delta)k_t^{NT}-\phi\left(\frac{i_t^{NT}}{k_t^{NT}}\right)k_t^{NT}\right)\\
&+\Psi_t^{B}\left(\begin{array}{c} a_t^{TR}f\left(k_t^{TR},N_t^{S,TR}\right)+a_t^{NT}g\left(k_t^{NT},N_t^{S,NT}\right)/p_t^{NT}- \\ \left(c_{ht}+c_{ht}^{*}+i_t^{TR}\right)-\left(d_t+i_t^{NT}\right)/p_t^{NT}-B_{t+1}-\left(1+R_t\right)B_t \end{array}\right)\\
&+\Psi_t^{NT}\left[a_t^{NT}\left(k_t^{NT}\right)^{1-\xi}\left(N_t^{S,NT}\right)^{\xi}-\left(d_t+i_t^{NT}\right)\right]\Bigg\}\\
&+\left(1-\pi\right)\left\{E_0\sum_{t=0}^{\infty}\Delta_t u\left(c_t^{TR*},c_t^{NT*},L_t^{*}\right)\right.\\
&+\ {}_t^{*}\left[c_t^{*}-\left(\omega_2\left(c_{ht}^{*}\right)^{-\mu}+\omega_1\left(c_{ft}^{*}\right)^{-\mu}\right)^{-\frac{1}{\mu}}\right]\\
&+\omega_t^{*}\left(1-L_t^{*}-N_t^{TR*}-N_t^{NT*}\right)\\
&+\Theta_t^{TR*}\left(LN_{t+1}^{S,TR*}-\zeta\left(\frac{N_t^{TR*}}{LN_t^{S,TR*}}\right)LN_t^{S,TR*}\right)\\
&+\Theta_t^{NT*}\left(LN_{t+1}^{S,NT*}-\zeta\left(\frac{N_t^{NT*}}{LN_t^{S,NT*}}\right)LN_t^{S,NT*}\right)\\
&+\Phi_t^{TR*}\left(LN_{t+1}^{S,TR*}-N_t^{S,TR*}\right)\\
&+\Phi_t^{NT*}\left(LN_{t+1}^{S,NT*}-N_t^{S,NT*}\right)\\
&+\lambda_t^{TR*}\left(\gamma k_{t+1}^{TR*}-(1-\delta)k_t^{TR*}-\phi\left(\frac{i_t^{TR*}}{k_t^{TR*}}\right)k_t^{TR*}\right)\\
&+\lambda_t^{NT*}\left(\gamma k_{t+1}^{NT*}-(1-\delta)k_t^{NT*}-\phi\left(\frac{i_t^{NT*}}{k_t^{NT*}}\right)k_t^{NT*}\right)\\
&+\Psi_t^{W}\left[a_t^{TR*}f\left(k_t^{TR*},N_t^{S,TR*}\right)-\left(c_{ht}+c_{ht}^{*}+i_t^{NT}\right)\right]\\
&+\Psi_t^{NT*}\left[a_t^{NT*}g\left(k_t^{NT*},N_t^{S,NT*}\right)-\left(d_t^{*}+i_t^{NT*}\right)\right]\Big\}\,.
\end{aligned}
$$

As before, the four resource constraints are combined into a world resource contraint after first order conditions are taken. Prices are determined in a manner similar to that in the baseline model.

References

[1] King, Robert and Mark Watson. "System Reduction and Solution algorithms for Singular Linear Difference Systems Under Rational Expectations." manuscript, Princeton University, 1995.

[2] ____________ and ____________. "The Solution of Singular Linear Difference Systems Under Rational Expectations." *International Economic Review 39* (1998), 1015-1026.